Influence of Pinned Joints on Damping and Dynamic Behavior of a Truss

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This paper discusses measured damping characteristics of a three-bay truss that uses pinned joints. A clevis/tang joint design was used. Pinned joints normally have a small amount of clearance between the pins and the clevis/tang fittings. The small gap can cause very significant changes in the dynamics and damping of a truss structure, particularly when the joint can traverse the deadband region. If the gravity induced loads prevent a joint from entering the deadband region, one might expect damping to be effected. Test results demonstrate that structures using pinned joints can have damping that is dependent on gravity. Damping rates can change by a factor of up to five due to variation of gravity induced loads. This variation could be very significant when characterizing space structures in a 1-g environment. The data also suggest that impact, as the joint gaps open and close, transfers vibration energy from low-frequency global modes to high-frequency local modes, increasing the rate of energy dissipation through material damping.

Introduction

TAH State University is currently developing an experiment titled joint damping experiment (JDX) to fly on the Space Shuttle as a Get Away Special (GAS) payload. The objective of this experiment is to measure the influence of gravity on the structural damping of a three-bay truss having clearance fit pinned joints. Structural damping is an important parameter in the dynamics of space structures. Future space structures will require more precise knowledge of structural damping than is currently available. JDX will allow researchers to characterize the influence of gravity and joint gap on structural damping and dynamic behavior of a small-scale truss. The data will be utilized in conjunction with nonlinear finite element analysis to determine which damping mechanisms are dominant.

Predicting damping in joints is quite difficult. One of the important variables affecting joint damping is gravity. Previous work has shown that gravity loads can influence damping in a pin-jointed truss structure. Flying this experiment as a GAS payload will allow testing in a microgravity environment. The on-orbit data (in microgravity) will be compared with ground test results.

This paper will document results of ground tests on an engineering model truss that was fabricated and tested to help refine a flight model truss design. It was desired to refine the truss design so that damping was significantly influenced by gravity loads. The mechanisms that produce damping in joints are not well understood. Several parameters that were suspected to influence damping were examined. Some of the variables of interest and reported in this paper include 1) joint stiffness and hysteresis, 2) joint gap, 3) truss assembly preloads, and 4) vacuum.

Models of Joint Damping

Models of joint or interface damping are derived from two mechanisms, friction and impacting. Friction is attributed to either rotary or extensional motions; impacting implies two surfaces are separated by some finite gap and come into contact during each cycle of an oscillation.

The method of energy dissipation for impacting is rather complicated. Crawley et al.² suggest that one measure of energy dissipation due to impacting is the coefficient of restitution. The coefficient of

restitution is not only a material property but is also a function of the shape of the contacting surfaces.

Models of friction damping generally fall into two categories, microslip and macroslip. Macroslip models assume no damping occurs until there is relative motion between two interfaces. Relative motion occurs when the forces parallel to the interface exceed Coulomb frictional force that is proportional to the force that is normal to the interface. This classic friction damping model was analyzed by Den Hartog.³ His analysis indicates that for small loads the energy dissipated increases linearly with displacement. Beards and Williams⁴ discuss how to optimize joint damping by maintaining an optimum joint load during rotational macroslip. They report that significant damping rates can be obtained when joints are allowed to undergo rotational slippage. Some static stiffness is sacrificed, however, when rotational slip is allowed.

Microslip models predict friction damping due to localized, microscopic slippage. Because of surface imperfections, interface contact pressure is not uniformly distributed. This allows localized slippage while the overall joint remains locked. For example, when material damping measurements are made using cantilever beam specimens, a prime concern is how the specimen is clamped to the wall such that microslip contributions are minimized. Damping due to microslip would typically be less than from macroslip. Plunkett⁵ reports that we are unable to predict damping due to microslip.

No general model is available describing losses in joints. However, analytical and computer models that attempt to include friction and impacting of a structure containing a single joint are available.^{6–9} Ferri⁹ showed computer simulations of the behavior of a single joint indicating that nonlinear sources of damping, such as friction and impacting, appear to be predominately viscous in nature. The models in the cited references, however, are not directly applicable to truss structures with multiple joints, although they could provide damping estimates of individual components.

Experiment Description

Figure 1 is a photograph of the JDX engineering model truss. The bottom bay of the truss is attached to a base plate that provides a cantilevered boundary condition for that end of the truss. A rigid end plate is attached to the top bay of the truss to provide a tip mass that lowers the natural frequencies. The excitation system can preferentially excite three modes in the truss, two bending modes and a torsional mode. The two bending modes are the two lowest frequency modes of the truss. These two bending modes are described in this paper as bend 1 and bend 2 modes. The lacing of the struts separates the two bending mode frequencies so each can be easily identified. The torsional mode consists of a rotational motion about

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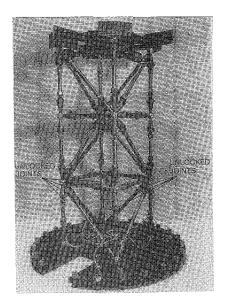


Fig. 1 Photograph of the truss.

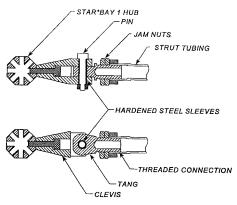


Fig. 2 Illustration of the design of the struts.

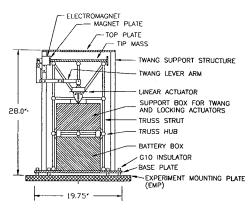


Fig. 3 Illustration of major components of the JDX experiment.

the long axis of the truss. The small plates attached to the tip mass in Fig. 1 are used by the excitation system.

Figure 2 illustrates the strut and joint design. The struts are fabricated from 6061-T6 aluminum. The strut length can be adjusted. The joints utilize press-fit hardened steel inserts to minimize joint wear. Hardened steel shoulder bolts are used for joint pins. The gap in the joints can be adjusted by using different diameter shoulder bolts. A joint can be converted to a locked joint by installing shims between the clevis and tang pieces and tightening the shoulder bolt. By applying high preloads with the shoulder bolts, the friction in this interface is sufficient to eliminate deadband, and joint rotations are prevented. By locking only a portion of the joints in the truss, a method of tailoring truss dynamics is obtained.

Requirements for GAS payloads have greatly influenced the design of this experiment. Figure 3 is an illustration showing some of

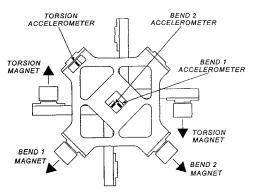


Fig. 4 Locations of accelerometers on the tip mass.

the major components that makeup the experiment. The experiment will be located inside a 5-ft³ GAS canister. The base plate of the experiment mounts to the GAS experiment mounting plate. Because GAS payloads must operate autonomously with their own power supply, a simple method of excitation and operation is required. A free decay or twang method of excitation is used. A twang test consists of displacing the truss tip a small distance and then releasing it. Damping is inferred from the transient decay.

The excitation assembly consists of a framework that supports linear actuators, linkages, and electromagnets. Part of the framework is illustrated in Fig. 3. The linear actuator shown in Fig. 3 is used to excite a bending mode. The actuator pushes on the lever arm that displaces an electromagnet horizontally. To excite a bending mode, the magnet is brought into contact with a plate attached with the tip mass and then power is applied to the magnet. Retracting the magnet will displace the truss in a shape resembling a bending mode. Removing the power to the electromagnet releases the tip mass. To excite the torsional mode, two magnets are operated simultaneously to provide a twisting motion to the truss. Figure 4 illustrates the direction the magnets move to excite the bend 1, bend 2, and torsion modes. The ground tests reported in this paper utilized the twang excitation hardware planned for flight.

To record the transient decay from a twang test, three accelerometers were attached to the tip mass as shown in Fig. 4. The accelerometers used were Kistler K-beams model 8302A2 with a 2-g full scale range. These accelerometers were selected for their high sensitivity. The accelerometers have a frequency response range from 0 to 400 Hz, which was felt to be adequate since the bending modes and the torsional mode are less than 100 Hz. The acceleration data were recorded at 3000 samples per second on all three channels.

Numerous twang tests were conducted to examine the effects of locking (or unlocking) various joints in the truss and selecting pin gap. It was shown that the truss is very sensitive to any preloads present in the struts. When strut lengths were set such that joint loads are minimized, the influence of the pinned joints on truss damping, frequency shift, and driving of higher vibration modes is maximized. It was desired to have near zero preloads in the struts with pinned joints. To monitor preloads during assembly, three strain gauges were attached to each strut at 120-deg intervals around each strut. Strains were monitored as the pin was inserted.

Preliminary tests showed that leaving only a few joints unlocked can significantly influence the dynamic behavior. The number of unlocked joints in the structure greatly adds to the difficulty in modeling the structure using a nonlinear finite element model. To simplify modeling, only eight joints were left unlocked in the truss. Figure 1 illustrates the locations of the unlocked joints. All eight unlocked joints are located on the truss at the same elevation. These joints were selected because they conveniently allow the truss to be partially disassembled.

To minimize the nonlinear effects of deadband, the largest pin diameter that could be inserted into the joint without inducing preloads was selected. This approach resulted in joints having slightly different pin gaps since the tolerance stack-up allows smaller pin gaps in some locations than others. The radial joint gap ranges from 0.0002 to 0.0004 in. in the engineering model truss. The truss appears tight to the touch with no easily detected deadband. A simple test for checking for joint preloads is whether joint pins can be freely

inserted and removed. If the joints are dry (unlubricated), the pin will not slide freely even though the strain gauge data indicate no preload. By lightly coating the pins with vacuum grease (Apiezon AP101), the pins slide freely when the preloads are very small. It is recognized that lubricating the pins can influence damping, particularly if friction is a significant mechanism in joint damping. It is felt, however, that pinned joints in space structures would typically be lubricated to ensure rotations can occur during deployment.

Strut Characterization

It was desired to characterize the stiffness and deadband in the truss struts. Simple tension/compression pull tests of the truss struts were conducted. During these tests, a strut experienced cyclic tension/compression loading. An MTS tensile testing machine provided a cyclic tensile load while three springs in parallel provided a compressive preload. A compliant link placed between the test fixture and the MTS allowed the cyclic tensile load to be applied in a very smooth, controlled fashion. A cyclic tensile load of 30–150 lb coupled with a compressive preload of 90 lb resulted in a ± 60 lb cycle applied to the test specimen. Displacements across one joint of the strut were measured using three linear variable displacement transducers (LVDTs). Load vs displacement data was recorded for several pin diameters and for a locked strut. Figure 5 illustrates the results.

The reduction in deadband with increasing pin diameter is clearly indicated in Fig. 5. Hysteresis loops in Fig. 5 demonstrate that friction forces are occurring as the joint traverses the deadband region. These friction forces could be a significant source of joint damping. The locked joint showed an essentially linear joint behavior with very little hysteresis. Since the loads in these hysteresis tests exceed those expected in twang tests of the truss, Fig. 5 indicates that the joints can remain locked. If slipping occurs in these locked joints, a new source of damping would be present. It was anticipated that the locked joints should have a small contribution to joint damping.

By integrating the area inside the hysteresis loops in Fig. 5, one can compute a loss factor for strut i (η_i) as

$$\eta_i = \Delta U_i / 2\pi U_i \tag{1}$$

where ΔU_i is the energy dissipated per cycle in strut i and U_i is the peak strain energy or the energy stored per cycle in the same strut. For built-up structures, an overall loss factor η_t is defined as

$$\eta_t = \left(\sum_{i=1}^n \Delta U_i \middle/ 2\pi \sum_{i=1}^n U_i\right) = \sum_{i=1}^n \eta_i f_i$$
(2)

where f_i is fraction of total strain energy in element i. Thus, if an appropriate strut loss factor can be determined, the damping

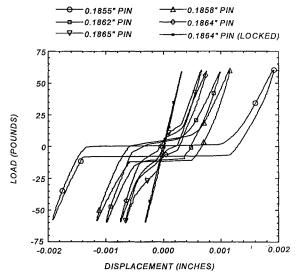


Fig. 5 Load vs displacement data from tension/compression tests of truss struts.

of a truss could be predicted. Unfortunately, the procedure used to measure the hysteresis loops in Fig. 5 are quasistatic. Damping mechanisms such as impacting are absent. A force-state mapping procedure has been reported as a good tool for measuring joint damping characteristics. ¹⁰ Efforts are currently underway to characterize the struts using this type of procedure.

Twang Test Results

Twang tests were conducted with the truss in two orientations with respect to the gravity vector. Figure 6 illustrates the 0- and 90-deg truss orientations. In the 0-deg orientation, gravity induced preloads in the struts are minimized, with the preload in the vertical struts being some fraction the weight of the truss (7.5 lb) and tip mass (15.5 lb) above it. Because it is very difficult to control the strut lengths exactly, the weight will not be uniformly distributed on the four vertical struts. A gravity induced preload between 2 and 8 lb was estimated to be present in the vertical struts in the 0-deg orientation. The 90-deg orientation provides the maximum gravity induced strut preloads. Two of struts attached to the base plate will have a 27-lb gravity induced preload in the 90-deg orientation.

During twang tests, the experiment was located inside a canister that was mounted to the floor providing the needed cantilever boundary condition. The canister allowed for testing in a vacuum or in air. Figure 7 illustrates the decay of the bend 1 mode for the truss with eight unlocked joints in a 0-deg orientation in both vacuum and air. The decays are essentially identical, and no significant change in damping is observed. Similar results were obtained for the bend 2 and torsion modes. The remainder of the paper will only present the results from tests done in air. All tests reported in this paper were conducted at room temperature ($20 \pm 2^{\circ}$ C).

Each twang test was repeated a minimum of five times to examine any scatter in the data. These repetitions produced nearly identical traces with a variation of about 0.002 g in amplitude and no significant changes in damping.

Figure 8 illustrates the acceleration time history of the bend 1 mode for the truss with all of the joints locked and the truss in a 0-deg orientation. A smooth, clear decay is observed with a relatively low damping rate. Figure 9 is the same plot over a shorter time scale. Figure 10 illustrates the decay of the bend 1 mode for the

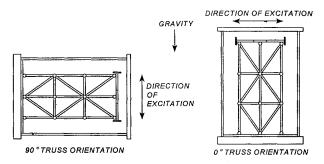


Fig. 6 Illustration of the 0- and 90-deg test orientations.

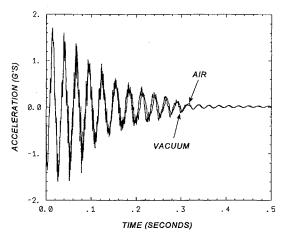


Fig. 7 Comparison of the decays of the bend 1 mode twang tests in air and vacuum at 0-deg truss orientation with eight unlocked joints.

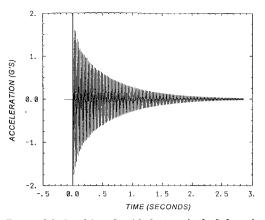


Fig. 8 Decay of the bend 1 mode with the truss in the 0-deg orientation and all joints locked.

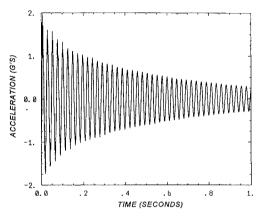


Fig. 9 Decay of the bend 1 mode with the truss in the 0-deg orientation and all joints locked (shortened time scale).

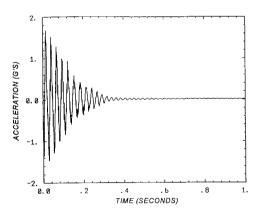


Fig. 10 Decay of the bend 1 mode with the truss in the 0-deg orientation and eight joints unlocked.

truss with eight unlocked joints and a 0-deg truss orientation. A dramatic increase in damping is observed in Fig. 10 when compared with Fig. 9.

Figure 11 illustrates the decay of the bend 1 mode with the truss in a 90-deg orientation with eight unlocked joints. The large gravity induced preloads in the 90-deg orientation can prevent deadband motion in the joints that carry most of the load. Deadband motion is possible in these joints only when the acceleration amplitude exceeds 1 g. Comparing Figs. 10 and 11, it is seen that damping decreases as strut preloads increase. Deadband motion also affects frequency. If deadband motion is prevented in a joint, the frequency of the mode should increase. The frequency of the bend 1 mode shifted from 36 Hz in the 0-deg orientation to 43 Hz in the 90-deg orientation.

Results from tests exciting the bend 2 mode produce the same net effect as the bend 1 mode. Torsion mode twang tests were obscured by a tip mass mode that was excited during the twang test. Changes

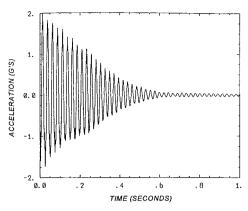


Fig. 11 Decay of the bend 1 mode with the truss in the 90-deg orientation and eight joints unlocked.

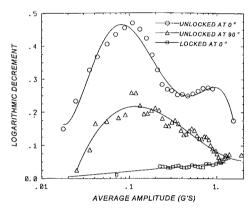


Fig. 12 Logarithmic decrement as a function of the average acceleration amplitude.

in the flight model tip mass design have been implemented to correct this problem in future testing.

Damping of the truss can be measured by the logarithmic decrement of a decay, assuming energy is not transferred to other modes. Figure 12 plots the logarithmic decrement of the bend 1 mode for the locked, 0- and 90-deg truss orientations as a function of acceleration amplitude. Figure 12 includes all five repetitions of each condition, and the symbols used to identify the individual curves represent a small fraction of the measured data points. The lines in the plot are curve fits to the data points. To allow examination of the influence of amplitude on damping, the logarithmic decrement is computed over a few cycles and is plotted against the average of the amplitudes used to compute the decay. Figure 12 shows that 1) damping rates can change by a factor of 2-5 as a result of simply changing the orientation of a truss, 2) the addition of a few unlocked joints to a truss structure can increase the damping by a factor as high as 40, and 3) damping is amplitude dependent. It is also interesting to note that at low amplitudes the damping in the 90-deg orientation approaches that of the locked truss. At low amplitudes, strut preloads may discourage a macroslip mode of friction damping as well as impacting. In this case, one would expect the damping to approach that of a locked joint. Although the damping becomes smaller at low amplitudes in the 0-deg truss orientation with unlocked joints, it is still significantly higher than the truss with locked joints.

A higher frequency signal or hash is observed in Figs. 10 and 11. Figures 13 and 14 show the first few cycles illustrated in Figs. 10 and 11. The hash is most pronounced in the 0-deg truss orientation with eight unlocked joints (Figs. 10 and 13). In the 90-deg orientation (Figs. 11 and 14), the hash mostly occurs around the $-1\,g$ amplitude level. At this amplitude, the truss is displaced upward such that the gravity preloads in the struts are removed and the joints can traverse the deadband zone. There is a correlation between joints entering the deadband and the occurrence of hash. The test results for the truss with all locked joints showed essentially no hash in its decay except for a small amount at the beginning of the decay. The observed hash is caused by higher modes of the

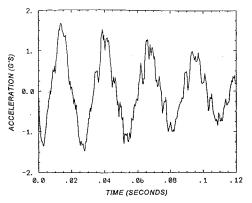


Fig. 13 Bend 1 mode decay for a 0-deg truss orientation and eight unlocked joints.

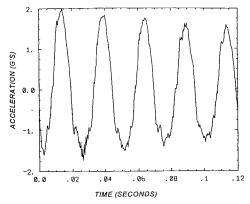


Fig. 14 Bend 1 mode decay for a 90-deg truss orientation and eight unlocked joints.

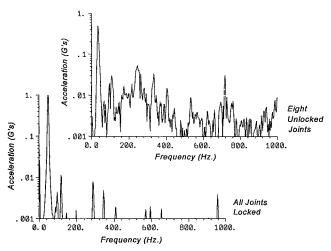


Fig. 15 Spectrum analyses of 0.3 s of a bend 1 mode test in the 0-deg orientation with locked joints (bottom) and eight unlocked joints (top).

truss and tip mass. The frequencies of the hash extends up through 1000 Hz. The amplitude of frequencies above 400 Hz, however, are in question because of the frequency response of the accelerometers. Figure 15 illustrates spectral analyses of the first 0.3 s in Figs. 9 and 10 for the locked and unlocked conditions, respectively. The comparison in Fig. 15 shows an order of magnitude increase in the amplitude of higher modes when unlocked joints are present as well as the broadband nature of the higher modes. Although these high-frequency modes can be excited by the twang excitation system, such high-frequency initial transients would rapidly decay, as observed in the test with locked joints. Thus, there is a clear indication that when a pinned joint traverses the deadband zone, vibration energy is transferred from the low-frequency global truss modes to the high-frequency truss modes where it is dissipated more rapidly by material damping. This form of energy dissipation in structures using pinned joints has been reported by others. 11,12 Impact damping could thus be described as the closure of a joint gap causing a step impulse to the surrounding framework. The magnitude of damping due to impact, however, might be significantly influenced by the nature of the framework attached to a joint. This may influence the accuracy of a force-state mapping procedure for determination of joint damping characteristics. The dynamics of a fixture used for holding a strut in a force-state mapping test might either encourage or discourage the rate of energy dissipation through impacting, as compared with behavior in a truss.

Although the test results indicated that impacting is a significant source of damping in pinned joints, friction is also suspected to provide a significant source of energy dissipation. The joints that carry the majority of the load in the 90-deg orientation tests with unlocked joints should not be traversing the deadband zone after the amplitude drops below 1 g. Lightly loaded joints in the truss could still be traversing the deadband zone, but the magnitude of the impacts ought to be small. Thus, much of the damping from the 90-deg tests with unlocked joints is suspected to be from a friction mechanism.

Analytical Modeling

It is anticipated that the behavior of the truss could be simulated by a nonlinear finite element model. The primary nonlinearity of the model is due to the gaps in the unlocked joints. As the joints go through their deadband region, the effective stiffness of the struts will change. Damping due to friction would also be a nonlinear effect. Linear models of the truss accurately predict the truss behavior with all joints locked. Nonlinear models are needed to model unlocked joints and to provide insight into damping mechanisms.

Commercial finite element programs such as MSC/NASTRAN¹³ have gap elements that allow modeling contacting surfaces. Previous efforts have been made to model a truss with pin joints using a detailed finite element model with numerous gap elements.¹⁴ This approach proved futile because of convergence problems in the solution procedure. Convergence problems occur when multiple gap elements can close or open in the same time step with significant changes in overall structure stiffness as a result. Thus it is important to utilize a simple model with the minimum number of gap elements although still sufficiently robust to capture the important mechanisms. A suggested approach is to first model an individual strut in a force-state mapping test where cyclic loading is applied by a small shaker and forces, accelerations, and displacements are measured. This approach will allow verification of stiffness, viscous damping, friction, and deadband characteristics of the strut. A finite element model of the entire truss would next be constructed.

A simple finite element model of an individual strut is illustrated in Figs. 16 and 17. In Fig. 16, beam elements are used to model the stiffness, mass, and material damping of the strut structural components. The unlocked joint is located between nodes 5 and 6 in Fig. 16. Figure 17 illustrates the finite element model of the unlocked joint. In Fig. 17, two gap elements (6 and 7) allow for axial deadband motion. Gap elements also allow modeling friction forces in a direction transverse to the gap direction. Elements 8 and 9 in Fig. 17 are always closed and provide a Coulomb friction force as the strut crosses the deadband region. Elements 8 and 9 are shown to be offset in the Y and Z directions, which are orthogonal to the strut orientation. Thus, energy dissipation due to friction could occur due

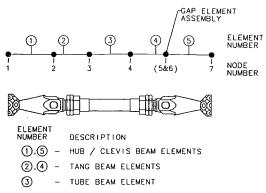


Fig. 16 Illustration of a finite element model of a strut.

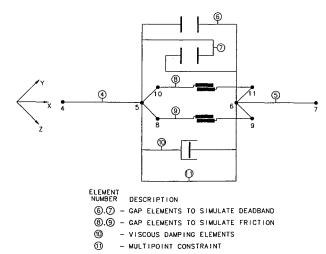


Fig. 17 Illustration of a finite element model of a strut joint.

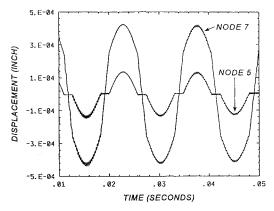


Fig. 18 Predicted time history for the simple strut model.

to both joint translations and rotations. Element 11 in Fig. 17 constrains the Y and Z displacement degrees of freedom of node 6 to equal those of node 5.

Analyses of the simple model in Figs. 16 and 17 have been completed. Quasistatic runs with this model can produce results very similar to the measured behavior in Fig. 5. Dynamic runs, however, can produce significant bouncing as the gap closes. That would indicate that impacting is an important mechanism in the joint behavior. A somewhat closer simulation of the anticipated behavior of the strut when installed in a truss can be obtained by including an additional element to the right of node 7. The added element approximates the stiffness of two additional struts and has a mass representative of the tip mass on the terminal node. Figure 18 illustrates the free decay of this simple model. The displacements predicted for node 5 clearly show the deadband regions. Note that each time the gap is closed, a higher frequency axial mode is induced in the model. Because of numerous assumptions about mass, stiffness terms, and friction and viscous damping coefficients that have yet to be validated, the results in Fig. 18 are only of qualitative value. Figure 18 does show that higher axial modes of the struts are excited by the impacting action of the joints. Force-state mapping tests of individual struts are currently underway, and the data will be used to validate the model. If reasonable simulation of individual strut behavior is obtained, then a three-dimensional model of the entire truss with eight unlocked joints will be generated. Results from the simulation of the truss model will be compared with the measured truss results.

Conclusions

A truss structure was fabricated and tested to examine the influence of joint gaps on the dynamics and damping. It was observed that inclusion of a few pin joints in the truss can greatly increase damping even though the joint gaps are small. Damping is maximized in such structures by minimizing strut preloads. Thus, damping of the JDX truss is very dependent on gravity induced loads. When deadband motion is allowed to occur in the truss, impacting in the joints was observed to drive higher modes in the truss, an effect that significantly increases damping. One consideration in strut characterization tests, such as force-state mapping, should be whether the test fixture might restrict or encourage energy transfer to higher modes because of the dynamic characteristics of the test fixture. Friction damping is also suspected to be a major contributor to the damping observed. A proposed method of modeling the truss using a nonlinear finite element model was discussed.

Acknowledgments

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